Published by Institute of Physics Publishing for SISSA

Received: *January 3, 2005*
Accepted: *May 27, 2005*
Published: *June 20, 2005*

A study of the Gribov copies in linear covariant gauges in euclidean Yang-Mills theories

Rodrigo F. Sobreiro and Silvio P. Sorella*

Departamento de Física Teórica, Instituto de Física
Universidade do Estado do Rio de Janeiro
Rua São Francisco Xavier 524, 20550-013 Maracanã, Rio de Janeiro, Brazil
E-mail: sobreiro@dft.if.uerj.br, sorella@uerj.br

Abstract: The Gribov copies and their consequences on the infrared behavior of the gluon propagator are investigated in euclidean Yang-Mills theories quantized in linear covariant gauges. Considering small values of the gauge parameter, it turns out that the transverse component of the gluon propagator is suppressed, while its longitudinal part is left unchanged. A Green function, $\mathcal{G}_{tr}(k)$, which displays infrared enhancement and which reduces to the ghost propagator in the Landau gauge is identified. The inclusion of the dimension two gluon condensate $\langle A_\mu^2\rangle$ is also considered. In this case, the transverse component of the gluon propagator and the Green function $\mathcal{G}_{tr}(k)$ remain suppressed and enhanced, respectively. Moreover, the longitudinal part of the gluon propagator becomes suppressed. A comparison with the results obtained from the studies of the Schwinger-Dyson equations and from lattice simulations is provided.

Keywords: Nonperturbative Effects, Confinement.

*Work supported by FAPERJ, Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro, under the program *Cientista do Nosso Estado*, E-26/151.947/2004.

Contents

1. Introduction

Gribov ambiguities [1] and their relevance for the nonperturbative aspects of euclidean Yang-Mills theories have witnessed growing interest in recent years. These ambiguities, affecting the Faddeev-Popov quantization formula, deeply modify the infrared behavior of Yang-Mills theories, as one learns from the large amount of results obtained in the Landau gauge [2−7] as well as in the Coulomb gauge [8−12].

As pointed out in [13], the existence of these ambiguities is due to the lack of a globally well defined gauge-fixing procedure. Among the class of covariant gauges, the Gribov ambiguities have been much investigated in the Landau gauge, where the gauge field is transverse, $\partial_\mu A_\mu^a = 0$. This property plays an important role here. It ensures that the Faddeev-Popov operator, $\mathcal{M}^{ab}(A) = -\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A_\mu^c\right)$, is hermitian, $\mathcal{M} = \mathcal{M}^\dagger$. Its eigenvalues are thus real. Concerning now the quantization of Yang-Mills theories in the Landau gauge, it turns out that, as a consequence of the existence of Gribov copies, the domain of integration in the Feynman path integral has to be restricted to the so called Gribov region Ω [1−7], which is the set of all transverse fields for which the Faddeev-Popov operator is positive definite, i.e. $\Omega = \left\{ A_\mu^a,\ \partial_\mu A_\mu^a = 0, \mathcal{M}^{ab}(A) = -\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A_\mu^c\right) > 0 \right\}$.

The boundary $\partial\Omega$, where the first vanishing eigenvalue of the Faddeev-Popov operator appears, is called the first Gribov horizon. For the partition function of Yang-Mills theories in the Landau gauge, one has

$$\begin{aligned}\mathcal{Z} &= \int_{\Omega} DA\delta(\partial A)\left(\det\left(-\partial^2\delta^{ab} + gf^{abc}A^c_\mu\partial_\mu\right)\right)e^{-\frac{1}{4}\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}} \\ &= \int_{\Omega} DAD\overline{c}Dc\delta(\partial A)e^{-\frac{1}{4}\int d^4x F^a_{\mu\nu}F^a_{\mu\nu} + \int d^4x\, \overline{c}^a\partial_\mu\left(\partial_\mu\delta^{ab} - gf^{abc}A^c_\mu\right)c^b}\,. \end{aligned} \tag{1.1}$$

The restriction of the domain of integration to the region Ω has important consequences on the infrared behavior of the gluon and ghost propagators [1−5, 7]. More precisely, in the Landau gauge, the gluon propagator $\left\langle A^a_\mu(k)A^b_\nu(-k)\right\rangle$ is found to be suppressed in the infrared, while the ghost propagator $\left\langle\overline{c}^a(k)c^b(-k)\right\rangle$ is enhanced, according to

$$\left\langle A^a_\mu(k)A^b_\nu(-k)\right\rangle = \delta^{ab}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{k^2}{k^4+\gamma^4}\,, \tag{1.2}$$

and

$$\begin{aligned}\mathcal{G}_{gh}(k) &= \frac{1}{N^2-1}\sum_{ab}\delta^{ab}\left\langle\overline{c}^a(k)c^b(-k)\right\rangle\,, \\ \mathcal{G}_{gh}(k)_{k\to 0} &\sim \frac{1}{k^4}\,. \end{aligned} \tag{1.3}$$

The Gribov parameter γ in eq. (1.2) has the dimension of a mass and is determined by a gap equation which, to the first order approximation, reads

$$\frac{3Ng^2}{4}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^4+\gamma^4} = 1\,. \tag{1.4}$$

From equation (1.3), one sees that the infrared behavior of the ghost propagator is more singular than the perturbative behavior $1/k^2$. This infrared enhancement is known as the Gribov-Zwanziger horizon condition [1, 4, 7], generally stated as $\lim_{k\to 0}\left(k^2\mathcal{G}_{gh}(k)\right)^{-1} = 0$. Remarkably, this behavior of the gluon and ghost propagators in the Landau gauge has received many confirmations from lattice simulations [14−22] as well as from recent studies of the Schwinger-Dyson equations [23−25, 27−30].

Several results have been established about the Gribov region Ω. It has been proven that Ω is convex and bounded in every direction [2, 5]. Moreover, every gauge orbit passes inside Ω [6]. The latter result is deeply related to the possibility of introducing an auxiliary functional, $\mathcal{F}(A) = \int d^4x A^a_\mu A^a_\mu$, whose minimization with respect to the gauge transformations provides a characterization of the region Ω [6, 31]. It can be checked in fact that the Gribov region Ω can be identified as the set of all relative minima of the functional $\mathcal{F}(A)$. It should also be mentioned here that the region Ω is not free from Gribov copies, i.e. Gribov copies still exist inside Ω [6, 31, 32]. To avoid the presence of these additional copies, a further restriction to a smaller region Λ, known as the fundamental modular region, should be implemented [6, 31, 32]. The region Λ is contained in the Gribov region Ω, being defined as the set of all absolute minima of the auxiliary functional $\mathcal{F}(A)$. However,

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it is difficult to give an explicit description of Λ. Recently, it has been argued in [33] that the additional copies existing inside Ω have no influence on the expectation values, so that averages calculated over Λ or Ω are expected to give the same result.

Besides the Landau gauge, the effects of the Gribov copies on the infrared behavior of Yang-Mills theories have been studied to a great extent in the noncovariant Coulomb gauge [8−12], $\partial_i A^a_i = 0$, $i = 1, 2, 3$. In particular, the Coulomb gauge allows a direct study of the confining properties of the static potential $V(r)$ between an external quark pair at spatial separation r. It turns out that $V(r)$ can be accessed by means of the 44-component, $\langle A^a_4(\overrightarrow{x}, t)A^b_4(0)\rangle$, of the gluon propagator. Moreover, in analogy with the Landau gauge, the spatial components of the gluon propagator, $\left\langle A^a_i(k)A^b_j(-k)\right\rangle$, are found to be suppressed in the infrared, a feature confirmed by lattice simulations [34, 35].

Concerning now other covariant gauges, although the presence of the Gribov copies is certainly to be expected [13], as explicitly confirmed by a recent study of the zero modes of the Faddeev-Popov operator in the maximal Abelian gauge [36], very little is known about their consequences on the infrared behavior of the gluon and ghost propagators and, more generally, on the whole Yang-Mills theory. The need for such an investigation is motivated by the increasing belief that the Gribov copies might have a crucial role for the infrared region of Yang-Mills theories as well as for color confinement. It would be desirable thus to improve our understanding on how the Gribov copies manifest themselves in different gauges and how they modify the infrared behavior of the theory.

This work aims at studying this issue in the class of covariant linear gauges, $\partial_\mu A^a_\mu = -\alpha b^a$, where α is the gauge parameter and b^a is the Lagrange multiplier. The task is far from being trivial. Many features of the Landau gauge are lost for a generic nonvanishing α. The Faddeev-Popov operator, $\mathcal{M}^{ab}(A) = -\partial_\mu\left(\partial_\mu\delta^{ab} - gf^{abc}A^c_\mu\right)$, is now not hermitian. Moreover, a suitable minimizing functional in these gauges is not yet at our disposal. As a consequence, the identification of the region to which the domain of integration in the path-integral should be restricted becomes difficult to be handled. All this makes a complete treatment of the Gribov copies in linear covariant gauges far beyond our present capabilities. Nevertheless, we shall be able to establish some preliminary results which allow us to obtain a characterization of the infrared behavior of the gluon propagator, at least for small values of the gauge parameter α. Considering in fact small values of the gauge parameter α, will allow us to stay close to the Landau gauge fixing. The present covariant gauge can be considered thus as a kind of deformation of the Landau gauge, whose results have to be recovered in the limit $\alpha \to 0$, thereby providing a useful consistency check.

The output of our findings can be summarized as follows. As in the case of the Landau gauge, the transverse component of the gluon propagator turns out to be suppressed in the infrared. Moreover, its longitudinal part remains unchanged.

Concerning now the behavior of the ghost fields in linear covariant gauges, it turns out that, instead of the ghost propagator, the Green function which is enhanced in the infrared is given by the quantity $\mathcal{G}_{tr}(k)$, defined as

$$\mathcal{G}_{tr}(k) = \frac{1}{N^2-1}\sum_{ab}\delta^{ab}\mathcal{G}^{ab}_{tr}(k)\,.$$

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$$\mathcal{G}^{ab}_{tr}(k) = \left\langle k \left| \left(\mathcal{M}^{-1}(A^T)\right)^{ab} \right| k \right\rangle . \tag{1.5}$$

where

$$A^{aT}_{\mu} = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A^a_\nu . \tag{1.6}$$

is the transverse component of the gauge field and $\left(\mathcal{M}^{-1}(A^T)\right)^{ab}$ stands for the inverse of the Faddeev-Popov operator $\mathcal{M}^{ab}(A^T)$,

$$\mathcal{M}^{ab}(A^T) = -\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^{Tc}_\mu \right) . \tag{1.7}$$

The Green function $\mathcal{G}_{tr}(k)$ is found to obey the Gribov-Zwanziger horizon condition, i.e. $\lim_{k\to 0} \left(k^2 \mathcal{G}_{tr}(k)\right)^{-1} = 0$. It should be remarked that $\mathcal{G}_{tr}(k)$ does not coincide with the ghost propagator for a generic value of the gauge parameter α. However, $\mathcal{G}_{tr}(k)$ reduces to the ghost two-point function for vanishing α, so that our results allow to recover those of the Landau gauge in the limit $\alpha \to 0$.

The infrared behavior of the gluon propagator and of $\mathcal{G}_{tr}(k)$ will be investigated also in the presence of the dimension two gluon condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$. A detailed study of this condensate in linear covariant gauges can be found in [37]. In the presence of $\left\langle A^a_\mu A^a_\mu \right\rangle$, the infrared suppression of the transverse component of the gluon propagator is enforced. Furthermore, its longitudinal component turns out to be suppressed as well. Concerning the Green function $\mathcal{G}_{tr}(k)$, its infrared enhancement is not modified by the condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$.

The work is organized as follows. In section 2 a few properties of the Gribov copies in the linear covariant gauges, and for small values of the gauge parameter α, are presented. In section 3 the infrared behavior of the gluon propagator, of the Green function $\mathcal{G}_{tr}(k)$ and of the ghost propagator is worked out. Section 4 accounts for the inclusion of the gluon condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$. Section 5 is devoted to a comparison of our results with those obtained from the analysis of the Schwinger-Dyson equations and from lattice simulations. In section 6 we present our conclusion.

2. A few properties of the Gribov copies in the linear covariant gauges

In this section we shall discuss a few properties of the Gribov copies in the linear covariant gauges. Let us begin by considering the expression of the euclidean Yang-Mills action quantized in the linear covariant $\alpha-$gauges, namely

$$S_{YM} + S_{gf} = \frac{1}{4} \int d^4x F^a_{\mu\nu} F^a_{\mu\nu} - \int d^4x \left(b^a \partial A^a + \frac{\alpha}{2} b^a b^a + \overline{c}^a \partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^c_\mu \right) c^b \right) . \tag{2.1}$$

where $\overline{c}^a$, c^a stand for the Faddeev-Popov ghosts. The Lagrange multiplier b^a has been introduced to implement the gauge condition

$$\partial_\mu A^a_\mu = -\alpha b^a . \tag{2.2}$$

from which it follows

$$\int d^4x \left(b^a \partial A^a + \frac{\alpha}{2} b^a b^a \right) \Rightarrow -\frac{1}{2\alpha} \int d^4x \ \left(\partial_\mu A^a_\mu \right)^2 . \tag{2.3}$$

From the relation (2.2) we see that the field A^a_μ is not transverse, due to the presence of the gauge parameter α. Of course, in the limit $\alpha \to 0$, we recover the Landau gauge, $\partial_\mu A^a_\mu = 0$. In what follows, it will be useful to decompose the gauge field A^a_μ into transverse and longitudinal components, according to

$$A^a_\mu = A^{aT}_\mu + A^{aL}_\mu \,. \tag{2.4}$$

with

$$\begin{aligned} A^{aT}_\mu &= \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right) A^a_\nu \,. \\ \partial_\mu A^{aT}_\mu &= 0 \,. \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} A^{aL}_\mu &= \frac{\partial_\mu\partial_\nu}{\partial^2} A^a_\nu = -\alpha \frac{\partial_\mu}{\partial^2} b^a \,. \\ \partial_\mu A^{aL}_\mu &= -\alpha b^a \,. \end{aligned} \tag{2.6}$$

As already underlined, we shall restrict ourselves to the case in which α is small, i.e. $\alpha \ll 1$, so that the longitudinal component A^{aL}_μ in eq. (2.4) can be considered as a small perturbation with respect to the transverse part A^{aT}_μ.

Let us proceed by proving the following statement.

Statement. *If the transverse component A^{aT}_μ of the gauge field $A^a_\mu = (A^{aT}_\mu + A^{aL}_\mu)$ belongs to the Gribov region Ω, i.e. $A^{aT}_\mu \in \Omega$, $\Omega = \{A^{aT}_\mu, \partial_\mu A^{aT}_\mu = 0, -\partial_\mu(\partial_\mu \delta^{ab} - g f^{abc} A^{cT}_\mu) > 0\}$, then the Faddeev-Popov operator $\mathcal{M}^{ab}(A) = -\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^c_\mu\right)$ has no zero modes.*

Proof. The proof of this statement is done by assuming the converse. Suppose indeed that the operator $\mathcal{M}^{ab}(A)$ has a zero mode, i.e. it exists a $\widetilde{\varphi}^a(x,\alpha)$ such that

$$-\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^{cT}_\mu - g f^{abc} A^{cL}_\mu\right) \widetilde{\varphi}^b(x,\alpha) = 0 \,. \tag{2.7}$$

which, from eq. (2.6), becomes

$$-\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^{cT}_\mu + \alpha g f^{abc} \left(\frac{\partial_\mu}{\partial^2} b^c\right)\right) \widetilde{\varphi}^b(x,\alpha) = 0 \,. \tag{2.8}$$

Since α is small we perform a Taylor expansion of $\widetilde{\varphi}^a(x,\alpha)$, namely

$$\widetilde{\varphi}^a(x,\alpha) = \sum_{n=0}^{\infty} \alpha^n \widetilde{\varphi}^a_n(x) \,. \tag{2.9}$$

with

$$\widetilde{\varphi}^a_0(x) = \left.\widetilde{\varphi}^a(x,\alpha)\right|_{\alpha=0} \,. \tag{2.10}$$

Equation (2.8) splits thus in a tower of equations, given by

$$-\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^{cT}_\mu\right)\widetilde{\varphi}^b_0(x)=0\,. \tag{2.11}$$

$$-\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^{cT}_\mu\right)\widetilde{\varphi}^b_1(x)-\partial_\mu\left(gf^{abc}\left(\frac{\partial_\mu}{\partial^2}b^c\right)\widetilde{\varphi}^b_0(x)\right)=0\,. \tag{2.12}$$

$$-\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^{cT}_\mu\right)\widetilde{\varphi}^b_2(x)-\partial_\mu\left(gf^{abc}\left(\frac{\partial_\mu}{\partial^2}b^c\right)\widetilde{\varphi}^b_1(x)\right)=0\,. \tag{2.13}$$

$$-\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^{cT}_\mu\right)\widetilde{\varphi}^b_3(x)-\partial_\mu\left(gf^{abc}\left(\frac{\partial_\mu}{\partial^2}b^c\right)\widetilde{\varphi}^b_2(x)\right)=0\,. \tag{2.14}$$

and so on.

Moreover, since A^{aT}_μ belongs to the Gribov region Ω, $A^{aT}_\mu\in\Omega$, it follows that $\widetilde{\varphi}^b_0(x)$ in the first equation (2.11) necessarily vanishes, i.e. $\widetilde{\varphi}^b_0(x)=0$, since the operator $-\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^{cT}_\mu\right)$ has no zero modes in Ω. Furthermore, setting $\widetilde{\varphi}^b_0(x)=0$ in the second equation (2.12), we get

$$-\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^{cT}_\mu\right)\widetilde{\varphi}^b_1(x)=0\,. \tag{2.15}$$

which, in turn, implies the vanishing of $\widetilde{\varphi}^b_1(x)$, i.e. $\widetilde{\varphi}^b_1(x)=0$. As a consequence, eq. (2.13) reads

$$-\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^{cT}_\mu\right)\widetilde{\varphi}^b_2(x)=0\,. \tag{2.16}$$

from which $\widetilde{\varphi}^b_2(x)=0$. It is apparent thus that the argument can be easily iterated to the whole tower of equations, implying that $\widetilde{\varphi}^b(x,\alpha)$ vanishes, $\widetilde{\varphi}^b(x,\alpha)=0$. This concludes the proof of the statement.

In particular, from this result it follows that if A^{aT}_μ belongs to the Gribov region, $A^{aT}_\mu\in\Omega$, the field $A^a_\mu=\left(A^{aT}_\mu+A^{aL}_\mu\right)$ does not possess Gribov copies which are closely related, i.e. obtained from A^a_μ through an infinitesimal gauge transformation

$$\widetilde{A}^a_\mu=A^a_\mu+\left(\partial_\mu\delta^{ab}-gf^{abc}A^c_\mu\right)\omega^b\,. \tag{2.17}$$

where $\omega^a(x)$ denotes the parameter of the gauge transformation. Indeed, from the condition

$$\partial_\mu\widetilde{A}^a_\mu=\partial_\mu A^a_\mu\,. \tag{2.18}$$

we should have

$$\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^c_\mu\right)\omega^b=0\,. \tag{2.19}$$

which, however, has no solution for $\omega^a(x)$.

2.1 Restriction of the domain of integration in the path-integral

The results obtained in the previous sections suggest to restrict the domain of integration in the path integral to the region $\widetilde{\Omega}$ defined as

$$\widetilde{\Omega}\equiv\left\{A^a_\mu;\ A^a_\mu=\ A^{aT}_\mu+A^{aL}_\mu,\ A^{aT}_\mu\in\Omega\right\}. \tag{2.20}$$

i.e. $\widetilde{\Omega}$ is the set of all connections whose transverse part A^{aT}_μ belongs to the Gribov region $\Omega=\left\{A^{aT}_\mu,\ \partial_\mu A^{aT}_\mu=0,\ -\partial_\mu\left(\partial_\mu\delta^{ab}-gf^{abc}A^{cT}_\mu\right)>0\right\}$. This choice is motivated by the following considerations:

- Since the gauge parameter α is small, $\alpha \ll 1$, the region $\widetilde{\Omega}$ can be regarded as a deformation of the Gribov region Ω. It is apparent in fact that in the limit $\alpha \to 0$ the region Ω is recovered, namely

$$\lim_{\alpha \to 0} \widetilde{\Omega} = \Omega \,. \tag{2.21}$$

- As discussed before, the Faddeev-Popov operator, $\mathcal{M}^{ab}(A) = -\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^c_\mu \right)$, has no zero modes within $\widetilde{\Omega}$. Therefore, the restriction to $\widetilde{\Omega}$ allows to get rid of the Gribov copies which can be obtained through infinitesimal gauge transformations.

- The effective implementation in the path-integral of the restriction of the domain of integration to the region $\widetilde{\Omega}$ can be done by repeating the no-pole prescription of Gribov's original work [1]. Indeed, since the transverse component A^{aT}_μ of any field belonging to $\widetilde{\Omega}$ lies in the Gribov region Ω, to implement the restriction to $\widetilde{\Omega}$ it will be sufficient to require that the Green function $\mathcal{G}_{tr}(k)$ of eq. (1.5) has no poles for any $A^{aT}_\mu \in \Omega$, except for a singularity at $k^2 = 0$, corresponding to the Gribov horizon $\partial\Omega$.

Thus, for the partition function of Yang-Mills theories in linear covariant gauges, we write

$$\begin{aligned} \mathcal{Z} &= \int DADb\delta(\partial A + \alpha b) \det \Big(\mathcal{M}^{ab}(A) \Big)\, e^{-\left(\frac{1}{4} \int d^4x F^a_{\mu\nu} F^a_{\mu\nu} - \int d^4x \left(b^a \partial A^a + \frac{\alpha}{2} b^a b^a \right) \right)} \mathcal{V}(\widetilde{\Omega}) \\ &= \int DA \, \det \Big(-\partial_\mu \left(\delta^{ab} \partial_\mu - g f^{abc} A^c_\mu \right) \Big)\, e^{-\left(\frac{1}{4} \int d^4x F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\alpha} \int d^4x (\partial A^a)^2 \right)} \mathcal{V}(\widetilde{\Omega}) \,. \end{aligned} \tag{2.22}$$

where the factor $\mathcal{V}(\widetilde{\Omega})$ implements the restriction to the region $\widetilde{\Omega}$. An explicit characterization of $\mathcal{V}(\widetilde{\Omega})$ and of its consequences on the infrared behavior of the gluon propagator and of $\mathcal{G}_{tr}(k)$ will be discussed in the next section.

Finally, it is worth to spend a few words on the aspects which remain uncovered by the present investigation. Even if the restriction to the region $\widetilde{\Omega}$ allows to get rid of the Gribov copies which are closely related, i.e. attainable by infinitesimal gauge transformations, we still lack a treatment of the copies which cannot be attained by infinitesimal transformations. This task is beyond our present possibilities, as the knowledge of a suitable auxiliary functional associated to the linear covariant gauges would be required. Nevertheless, since we are limiting ourselves to small values of α, the restriction to the region $\widetilde{\Omega}$ looks quite natural.

2.2 Characterization of the factor $\mathcal{V}(\widetilde{\Omega})$

As already remarked, the factor $\mathcal{V}(\widetilde{\Omega})$ can be accommodated for by requiring that the Green function $\mathcal{G}_{tr}(k)$ of eq. (1.5) has no poles for a given nonvanishing value of the momentum k. This condition can be understood by recalling that the region Ω is defined as the set of all transverse gauge connections $\left\{ A^{Ta}_\mu \right\}$, $\partial_\mu A^{Ta}_\mu = 0$, for which the operator $\mathcal{M}^{ab}(A^T)$ is positive definite, i.e. $\mathcal{M}^{ab}(A^T) = -\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^{Tc}_\mu \right) > 0$. This implies that the inverse operator $\left[-\partial_\mu \left(\partial_\mu \delta^{ab} - g f^{abc} A^{Tc}_\mu \right) \right]^{-1}$, and thus $\mathcal{G}_{tr}(k)$, can become large only when approaching the horizon $\partial\Omega$, which corresponds in fact to $k = 0$ [1]. The quantity $\mathcal{G}_{tr}(k)$

can be evaluated order by order in perturbation theory. Repeating the same calculation of [1], we find that, up to the second order

$$\mathcal{G}_{tr}(k) \approx \frac{1}{k^2} \frac{1}{1-\rho(k,A^T)} \,. \tag{2.23}$$

with $\rho(k, A^T)$ given by

$$\begin{aligned} \rho(k,A^T) &= \frac{N}{N^2-1} \frac{g^2}{V} \frac{1}{k^2} \sum_q \frac{k_\mu (k-q)_\nu}{(k-q)^2} A^{Ta}_\mu(q) A^{Ta}_\nu(-q) \\ &= \frac{N}{N^2-1} \frac{g^2}{V} \frac{k_\mu k_\nu}{k^2} \sum_q \frac{1}{(k-q)^2} A^{Ta}_\mu(q) A^{Ta}_\nu(-q) \,. \end{aligned} \tag{2.24}$$

V being the euclidean space-time volume. According to [1], the no-pole condition for $\mathcal{G}_{tr}(k)$ reads

$$\begin{aligned} \rho(0,A^T) &< 1 \,. \\ \rho(0,A^T) &= \frac{1}{4} \frac{N}{N^2-1} \frac{g^2}{V} \sum_q \frac{1}{q^2} \left(A^{Ta}_\lambda(q) A^{Ta}_\lambda(-q) \right) . \end{aligned} \tag{2.25}$$

Therefore, for the factor $\mathcal{V}(\widetilde{\Omega})$ in eq. (2.22) we have

$$\mathcal{V}(\widetilde{\Omega}) = \theta \left(1 - \rho(0,A^T) \right) \,. \tag{2.26}$$

where $\theta(x)$ stands for the step function.[1]

3. The gluon propagator

In order to study the gluon propagator, it is sufficient to retain only the quadratic terms in expression (2.22) which contribute to the two-point correlation function $\langle A^a_\mu(k) A^b_\nu(-k) \rangle$. Making use of the integral representation for the step function

$$\theta \left(1 - \rho(0,A^T) \right) = \int_{-i\infty+\varepsilon}^{i\infty+\varepsilon} \frac{d\eta}{2\pi i\eta} e^{\eta\left(1-\rho\left(0,A^T\right)\right)} \,. \tag{3.1}$$

for the partition function (2.22) one get

$$\mathcal{Z}_{\rm quadr} = \mathcal{N} \int DA \frac{d\eta}{2\pi i} e^{\eta - \log\eta} e^{-\left(S_{\rm quadr} + \eta\rho(0,A^T)\right)} . \tag{3.2}$$

where $\mathcal{N}$ is a constant factor and $S_{\rm quadr}$ stands for the quadratic part of the quantized Yang-Mills action

$$S_{\rm quadr} = \frac{1}{2} \sum_q \left(q^2 A^a_\mu(q) A^a_\mu(-q) - \left(1 - \frac{1}{\alpha} \right) q_\mu q_\nu A^a_\mu(q) A^a_\nu(-q) \right) . \tag{3.3}$$

[1] $\theta(x) = 1$ if $x > 0$, $\theta(x) = 0$ if $x < 0$.

From

$$A_\mu^{Ta}(q)A_\mu^{Ta}(-q) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left(\delta_{\mu\sigma} - \frac{q_\mu q_\sigma}{q^2}\right) A_\nu^a(q)A_\sigma^a(-q)$$
$$= A_\mu^a(q)A_\mu^a(-q) - \frac{q_\mu q_\nu}{q^2} A_\mu^a(q)A_\nu^a(-q)\,. \tag{3.4}$$

it follows that

$$\mathcal{Z}_{\text{quadr}} = \mathcal{N}\int DA\frac{d\eta}{2\pi i}e^{\eta-\log\eta}e^{-\frac{1}{2}\sum_q A_\mu^a(q)\mathcal{Q}_{\mu\nu}^{ab}A_\nu^b(-q)}\,. \tag{3.5}$$

with

$$\mathcal{Q}_{\mu\nu}^{ab} = \left(\left(q^2 + \frac{\eta N g^2}{N^2-1}\frac{1}{2V}\frac{1}{q^2}\right)\delta_{\mu\nu} - q_\mu q_\nu\left(\left(1-\frac{1}{\alpha}\right) + \frac{\eta N g^2}{N^2-1}\frac{1}{2V}\frac{1}{q^4}\right)\right)\delta^{ab}\,. \tag{3.6}$$

Integrating over the gauge field, one has

$$\mathcal{Z}_{\text{quadr}} = \mathcal{N}\int \frac{d\eta}{2\pi i}e^{\eta-\log\eta}\left(\det \mathcal{Q}_{\mu\nu}^{ab}\right)^{-\frac{1}{2}} = \mathcal{N}'\int\frac{d\eta}{2\pi i}e^{f(\eta)}\,. \tag{3.7}$$

where $f(\eta)$ is given by

$$f(\eta) = \eta - \log\eta - \frac{3}{2}(N^2-1)\sum_q \log\left(q^4 + \frac{\eta N g^2}{N^2-1}\frac{1}{2V}\right)\,. \tag{3.8}$$

Following [1], the expression (3.7) can be now evaluated at the saddle point, namely

$$\mathcal{Z}_{\text{quadr}} \approx e^{f(\eta_0)}\,. \tag{3.9}$$

where η_0 is determined by the minimum condition

$$1 - \frac{1}{\eta_0} - \frac{3}{4}\frac{Ng^2}{V}\sum_q\frac{1}{\left(q^4 + \frac{\eta_0 N g^2}{N^2-1}\frac{1}{2V}\right)} = 0\,. \tag{3.10}$$

Taking the thermodynamic limit, $V\to\infty$, and introducing the Gribov parameter γ [1]

$$\gamma^4 = \frac{\eta_0 N g^2}{N^2-1}\frac{1}{2V}\,. \qquad V\to\infty\,. \tag{3.11}$$

we get the gap equation

$$\frac{3}{4}Ng^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^4+\gamma^4} = 1\,. \tag{3.12}$$

where the term $1/\eta_0$ in (3.10) has been neglected in the thermodynamic limit. To obtain the gauge propagator, we can now go back to the expression for $\mathcal{Z}_{\text{quadr}}$ which, after substituting the saddle point value $\eta = \eta_0$, becomes

$$\mathcal{Z}_{\text{quadr}} = \mathcal{N}\int DA e^{-\frac{1}{2}\sum_q A_\mu^a(q)\mathcal{Q}_{\mu\nu}^{ab}A_\nu^b(-q)}\,. \tag{3.13}$$

with

$$\mathcal{Q}^{ab}_{\mu\nu} = \left(\left(q^2+\frac{\gamma^4}{q^2}\right)\delta_{\mu\nu} - q_\mu q_\nu\left(\left(1-\frac{1}{\alpha}\right)+\frac{\gamma^4}{q^4}\right)\right)\delta^{ab}$$
$$= \left(\left(q^2+\frac{\gamma^4}{q^2}\right)\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)+\frac{q_\mu q_\nu}{q^2}\left(\frac{q^2}{\alpha}\right)\right)\delta^{ab}. \tag{3.14}$$

Evaluating the inverse of $\mathcal{Q}^{ab}_{\mu\nu}$, we get the gluon propagator

$$\left\langle A^a_\mu(q)A^b_\nu(-q)\right\rangle = \delta^{ab}\left(\frac{q^2}{q^4+\gamma^4}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)+\frac{\alpha}{q^2}\frac{q_\mu q_\nu}{q^2}\right). \tag{3.15}$$

A few remarks are now in order.

Let us first observe that the gap equation (3.12) defining the parameter γ has the same form of that obtained by Gribov in the Landau gauge [1]. This is an expected result, since the factor $\rho(0,A^T)$ in equation (2.25) depends only on the transverse component A^{Ta}_μ.

As it is apparent from the expression (3.15), the transverse component of the gluon propagator is suppressed in the infrared, while the longitudinal component is left unchanged. Moreover, as we shall see later, the behavior of the longitudinal part turns out to be modified once the gluon condensate $\left\langle A^a_\mu A^a_\mu\right\rangle$ is taken into account.

Finally, in the limit $\alpha\to 0$, the results of the Landau gauge are recovered.

3.1 The infrared behavior of $\mathcal{G}_{tr}(k)$

Let us discuss now the infrared behavior of the Green function $\mathcal{G}_{tr}(k)$ of eq. (1.5), which is obtained upon contraction of the gauge fields in the expression (2.24), namely

$$\mathcal{G}_{tr}(k) \approx \frac{1}{k^2}\frac{1}{1-\rho(k)}. \tag{3.16}$$

with

$$\rho(k) = g^2\frac{N}{N^2-1}\frac{k_\mu k_\nu}{k^2}\int\frac{d^4q}{(2\pi)^4}\frac{1}{(k-q)^2}\left\langle A^{Ta}_\mu(q)A^{Ta}_\nu(-q)\right\rangle. \tag{3.17}$$

From the expression of the gluon propagator in eq. (3.15), it follows

$$\rho(k) = g^2N\frac{k_\mu k_\nu}{k^2}\int\frac{d^4q}{(2\pi)^4}\frac{1}{(k-q)^2}\frac{q^2}{q^4+\gamma^4}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right). \tag{3.18}$$

Furthermore, from the gap equation (3.12), it turns out

$$Ng^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^4+\gamma^4}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right) = \delta_{\mu\nu}. \tag{3.19}$$

so that

$$1-\rho(k) = Ng^2\frac{k_\mu k_\nu}{k^2}\int\frac{d^4q}{(2\pi)^4}\frac{k^2-2qk}{(k-q)^2}\frac{1}{q^4+\gamma^4}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right). \tag{3.20}$$

Notice that the integral in the right hand side of eq. (3.20) is convergent and nonsingular at $k=0$. Therefore, for $k\approx 0$,

$$\left(1-\rho(k)\right)_{k\approx 0} \approx \frac{3Ng^2\mathcal{I}}{4}k^2. \tag{3.21}$$

where $\mathcal{I}$ stands for the value of the integral

$$\mathcal{I} = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2(q^4+\gamma^4)} = \frac{1}{32\pi} \frac{1}{\gamma^2} \,. \tag{3.22}$$

Finally, for the infrared behavior of the Green function $\mathcal{G}_{tr}(k)$ we get

$$(\mathcal{G}_{tr})_{k\approx 0} \approx \frac{4}{3Ng^2\mathcal{I}} \frac{1}{k^4} \,. \tag{3.23}$$

One sees thus that $\mathcal{G}_{tr}(k)$ is enhanced in the infrared, obeying the Gribov-Zwanziger condition $\lim_{k\to 0} \left(k^2\mathcal{G}_{tr}(k)\right)^{-1} = 0$.

3.2 Analysis of the ghost propagator

For the sake of completeness, let us discuss here the infrared behavior of the ghost two-point function, $\mathcal{G}_{gh}(k)$, given by

$$\mathcal{G}_{gh}(k) = \frac{1}{N^2-1} \sum_{ab} \delta^{ab} \left\langle \overline{c}^a(k) c^b(-k) \right\rangle \approx \frac{1}{k^2} \frac{1}{1-\omega(k)} \,. \tag{3.24}$$

with

$$\omega(k) = \frac{N}{N^2-1} \frac{g^2}{k^2} \int \frac{d^4q}{(2\pi)^4} \frac{k_\mu (k-q)_\nu}{(k-q)^2} \left\langle A^a_\mu(q) A^a_\nu(-q) \right\rangle \,. \tag{3.25}$$

Making use of the expression for the gluon propagator in eq. (3.15) and of the equation (3.19), it follows that, in the infrared,

$$1-\omega(k) \approx \frac{3Ng^2}{128\pi} \frac{k^2}{\gamma^2} - \frac{\alpha N g^2}{k^2} \int \frac{d^4q}{(2\pi)^4} \frac{k_\mu (k-q)_\nu}{(k-q)^2} \frac{q_\mu q_\nu}{q^4} \,. \tag{3.26}$$

The second term in the right hand-side of eq. (3.26) can be easily evaluated by means of the dimensional regularization. Adopting the $\overline{MS}$ scheme, the final expression for the factor $(1-\omega(k))$ is found

$$(1-\omega(k)) \approx \frac{3Ng^2}{128\pi} \frac{k^2}{\gamma^2} - \frac{\alpha N g^2}{64\pi^2} \log \frac{k^2}{\overline{\mu}^2} \,. \tag{3.27}$$

One sees that, in the present case, the Gribov-Zwanziger horizon condition is jeopardized by the term $\log \frac{k^2}{\overline{\mu}^2}$ in expression (3.27). As it is apparent from the presence of the gauge parameter α, this term is due to the contribution of the longitudinal components of the gauge field. Notice also that the longitudinal modes do not contribute to the Green function $\mathcal{G}_{tr}(k)$ in eq. (3.16).

4. Inclusion of the dimension two condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$

The dimension two gluon condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$ has received much attention in the last years [38–48]. This condensate turns out to contribute to the gluon two-point function, as observed in [49] within the operator product expansion. As such, it has to be taken into account when discussing the gluon propagator. A renormalizable effective potential for $\left\langle A^a_\mu A^a_\mu \right\rangle$ in linear covariant gauges has been constructed and evaluated in analytic form

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in [37]. The output of this investigation is that a nonvanishing value of the condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$ is favoured since it lowers the vacuum energy. As a consequence, a dynamical tree level gluon mass is generated in the gauge fixed Lagrangian [37]. The inclusion of the condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$ in the present framework can be done along the lines outlined in [50], where the effects of the Gribov copies on the gluon and ghost propagators in the presence of $\left\langle A^a_\mu A^a_\mu \right\rangle$ have been worked out in the Landau gauge. Let us begin by giving a brief account of the dynamical mass generation in linear covariant gauges. Following [37], the dynamical mass generation in these gauges is described by the following action

$$S(A,\sigma) = S_{YM} + S_{gf} + S_\sigma \,. \tag{4.1}$$

where S_{YM}, S_{gf} are the Yang-Mills and the gauge fixing terms, as given in eq. (2.1). The term S_σ in eq. (4.1) contains the auxiliary scalar field σ and reads

$$S_\sigma = \int d^4x \left(\frac{\sigma^2}{2g^2\zeta} + \frac{1}{2}\frac{\sigma}{g\zeta} A^a_\mu A^a_\mu + \frac{1}{8\zeta}\left(A^a_\mu A^a_\mu\right)^2 \right). \tag{4.2}$$

The introduction of the auxiliary field σ allows to study the condensation of the local operator $A^a_\mu A^a_\mu$. In fact, as shown in [37], the following relation holds

$$\langle \sigma \rangle = -\frac{g}{2}\left\langle A^a_\mu A^a_\mu \right\rangle \,. \tag{4.3}$$

The dimensionless parameter ζ in expression (4.2) is needed to account for the ultraviolet divergences present in the vacuum correlation function $\left\langle A^2(x) A^2(y) \right\rangle$. For the details of the renormalizability properties of the local operator $A^a_\mu A^a_\mu$ in linear covariant gauges we refer to [51]. The action $S(A,\sigma)$ is the starting point for evaluating the renormalizable effective potential $V(\sigma)$ for the auxiliary field σ, obeying the renormalization group equations. The minimum of $V(\sigma)$ occurs for a nonvanishing vacuum expectation value of the auxiliary field [37], i.e. $\langle \sigma \rangle \neq 0$. In particular, from expression (4.1), the first order induced dynamical gluon mass is found to be

$$m^2 = \frac{g\,\langle \sigma \rangle}{\zeta_0} \,. \tag{4.4}$$

where ζ_0 is the first contribution to the parameter ζ, given by [37]

$$\begin{aligned} \zeta &= \frac{\zeta_0}{g^2} + \zeta_1 + O(g^2) \,. \\ \zeta_0 &= \frac{3\left(78 - 26\alpha^2 + 3\alpha^3 + 18\alpha\log\alpha\right)}{2\left(3\alpha - 13\right)^2}\frac{\left(N^2-1\right)}{N} \,. \end{aligned} \tag{4.5}$$

We remind here that in linear covariant gauges, the Faddeev-Popov ghosts $\overline{c}^a$, c^a remain massless, due to the absence of mixing between the composite operators $A^a_\mu(x) A^a_\mu(x)$ and $\overline{c}^a(x) c^a(x)$. This stems from additional Ward identities present in these gauges [51], which forbid the appearance of the term $\overline{c}^a(x) c^a(x)$.

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4.1 Infrared behavior of the gluon propagator in the presence of $\langle A^a_\mu A^a_\mu \rangle$

It is worth underlining that the action $S(A,\sigma)$ leads to a partition function which is still plagued by the Gribov copies. It might be useful to notice indeed that the action $(S_{YM}+S_\sigma)$ is left invariant by the local gauge transformations

$$\begin{aligned} \delta A^a_\mu &= -D^{ab}_\mu \omega^b \,. \\ \delta\sigma &= g A^a_\mu \partial_\mu \omega^a \,. \end{aligned} \tag{4.6}$$

$$\delta\left(S_{YM}+S_\sigma\right)=0\,. \tag{4.7}$$

Therefore, implementing the restriction to the region $\widetilde{\Omega}$, for the partition function we obtain

$$\mathcal{Z}=\int DA\, D\sigma \det\left(-\partial_\mu\left(\delta^{ab}\partial_\mu+gf^{abc}A^c_\mu\right)\right) e^{-\left(\frac{1}{4}\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}+\frac{1}{2\alpha}\int d^4x(\partial A^a)^2+S_\sigma\right)}\mathcal{V}(\widetilde{\Omega})\,. \tag{4.8}$$

with the factor $\mathcal{V}(\widetilde{\Omega})$ given in eqs. (2.25),(2.26). To discuss the gluon propagator we proceed as before and retain only the quadratic terms in expression (4.8) which contribute to the two-point correlation function $\langle A^a_\mu(k)A^b_\nu(-k)\rangle$. Expanding around the nonvanishing vacuum expectation value of the auxiliary field, $\langle\sigma\rangle\neq 0$, one easily get

$$\begin{aligned} \mathcal{Z}_{\rm quadr} &= \mathcal{N}\int DA\frac{d\eta}{2\pi i\eta}e^{\eta(1-\rho(0,A))}e^{-\left(\frac{1}{4}\int d^4x((\partial_\mu A^a_\nu-\partial_\mu A^a_\nu)^2+\frac{1}{2\alpha}\int d^4x(\partial A^a)^2+\frac{1}{2}m^2\int d^4x\left(A^a_\mu A^a_\mu\right)\right)} \\ &= \mathcal{N}\int DA\frac{d\eta}{2\pi i}e^{\eta-\log\eta}e^{-\frac{1}{2}\sum_q A^a_\mu(q)\mathcal{Q}^{ab}_{\mu\nu}A^b_\nu(-q)}\,. \end{aligned} \tag{4.9}$$

where the factor $\mathcal{Q}^{ab}_{\mu\nu}$ is now given by

$$\mathcal{Q}^{ab}_{\mu\nu}=\left(\left(q^2+m^2+\frac{\eta Ng^2}{N^2-1}\frac{1}{2V}\frac{1}{q^2}\right)\delta_{\mu\nu}-q_\mu q_\nu\left(\left(1-\frac{1}{\alpha}\right)+\frac{\eta Ng^2}{N^2-1}\frac{1}{2V}\frac{1}{q^4}\right)\right)\delta^{ab}\,. \tag{4.10}$$

Integrating over the gauge field, one has

$$\mathcal{Z}_{\rm quadr}=\mathcal{N}\int\frac{d\eta}{2\pi i}e^{\eta-\log\eta}\left(\det\mathcal{Q}^{ab}_{\mu\nu}\right)^{-\frac{1}{2}}=\mathcal{N}'\int\frac{d\eta}{2\pi i}e^{f(\eta)}\,. \tag{4.11}$$

with

$$f(\eta)=\eta-\log\eta-\frac{3}{2}(N^2-1)\sum_q\log\left(q^4+m^2q^2+\frac{\eta Ng^2}{N^2-1}\frac{1}{2V}\right)\,. \tag{4.12}$$

Evaluating $\mathcal{Z}_{\rm quadr}$ at the saddle point, yields

$$\mathcal{Z}_{\rm quadr}\approx e^{f(\eta_0)}\,. \tag{4.13}$$

where η_0 is determined by the minimum condition

$$1-\frac{1}{\eta_0}-\frac{3}{4}\frac{Ng^2}{V}\sum_q\frac{1}{\left(q^4+m^2q^2+\frac{\eta_0 Ng^2}{N^2-1}\frac{1}{2V}\right)}=0\,. \tag{4.14}$$

Taking the thermodynamic limit, $V \to \infty$, and introducing the Gribov parameter

$$\gamma^4 = \frac{\eta_0 N g^2}{N^2-1}\frac{1}{2V} \, . \qquad V \to \infty \, . \tag{4.15}$$

we get the gap equation in the presence of the dynamical gluon mass, corresponding to a nonvanishing condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$, namely

$$\frac{3}{4} N g^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^4 + m^2 q^2 + \gamma^4} = 1 \, . \tag{4.16}$$

Notice that the dynamical mass m appears now explicitly in the gap equation (4.16). To obtain the gauge propagator, one goes back to the expression (4.9) which, when evaluated at the saddle point value $\eta = \eta_0$, yields

$$\mathcal{Z}_{\mathrm{quadr}} = \mathcal{N} \int DA e^{-\frac{1}{2}\sum_q A^a_\mu(q) \mathcal{Q}^{ab}_{\mu\nu} A^b_\nu(-q)} \, . \tag{4.17}$$

with

$$\mathcal{Q}^{ab}_{\mu\nu} = \left(\left(q^2 + m^2 + \frac{\gamma^4}{q^2} \right) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{q_\mu q_\nu}{q^2} \left(\frac{q^2}{\alpha} + m^2 \right) \right) \delta^{ab} \, . \tag{4.18}$$

Thus, for the gauge propagator in the presence of the dynamical gluon mass m we get

$$\left\langle A^a_\mu(q) A^b_\nu(-q) \right\rangle = \delta^{ab} \left(\frac{q^2}{q^4 + m^2 q^2 + \gamma^4} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{\alpha}{q^2 + \alpha m^2} \frac{q_\mu q_\nu}{q^2} \right) . \tag{4.19}$$

We notice that, due to the presence of the mass m, the infrared suppression of the transverse component of the gluon propagator is enforced. Moreover, also the longitudinal component gets suppressed.

4.2 The infrared behavior of $\mathcal{G}_{tr}(k)$ in the presence of $\left\langle A^a_\mu A^a_\mu \right\rangle$

It remains now to discuss the infrared behavior of the Green function $\mathcal{G}_{tr}(k)$ in the presence of $\left\langle A^a_\mu A^a_\mu \right\rangle$. This can be easily worked out by repeating the analysis done in the previous sections. From the expression of the gluon propagator (4.19), it follows that

$$\mathcal{G}_{tr}(k) \approx \frac{1}{k^2} \frac{1}{1 - \rho(k)} \, . \tag{4.20}$$

with

$$\begin{aligned} \rho(k) &= g^2 \frac{N}{N^2-1} \frac{k_\mu k_\nu}{k^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(k-q)^2} \left\langle A^{Ta}_\mu(q) A^{Ta}_\nu(-q) \right\rangle \\ &= g^2 N \frac{k_\mu k_\nu}{k^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(k-q)^2} \frac{q^2}{q^4 + m^2 q^2 + \gamma^4} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) . \end{aligned} \tag{4.21}$$

Also, from the gap equation (4.16), one has

$$N g^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^4 + m^2 q^2 + \gamma^4} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) = \delta_{\mu\nu} \, . \tag{4.22}$$

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so that

$$1-\rho(k) = Ng^2 \frac{k_\mu k_\nu}{k^2} \int \frac{d^4q}{(2\pi)^4} \frac{k^2-2qk}{(k-q)^2} \frac{1}{q^4+m^2q^2+\gamma^4} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \tag{4.23}$$

Thus, for $k \approx 0$,

$$(1-\rho(k))_{k\approx 0} \approx \frac{3Ng^2\mathcal{J}}{4} k^2 . \tag{4.24}$$

where $\mathcal{J}$ stands for the value of the integral

$$\mathcal{J} = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2(q^4+m^2q^2+\gamma^4)} . \tag{4.25}$$

which is ultraviolet finite. Therefore, for the Green function $\mathcal{G}_{tr}(k)$, we get

$$(\mathcal{G}_{tr}(k))_{k\approx 0} \approx \frac{4}{3Ng^2\mathcal{J}} \frac{1}{k^4} . \tag{4.26}$$

exhibiting the infrared enhancement which, thanks to the gap equation (4.16), turns out to hold also in the presence of the gluon condensate $\langle A^a_\mu A^a_\mu \rangle$.

5. Comparison with the results obtained from lattice simulations and from the Schwinger-Dyson equations

Having investigated the infrared behavior of the gluon propagator and of the Green function $(\mathcal{G}_{tr}(k))$, as summarized by equations (4.19) and (4.26), it is useful to make a comparison with the results already available from lattice simulations and from the studies of the Schwinger-Dysons equations. Let us begin with the lattice data

5.1 Comparison with the lattice data

In a series of papers [52 – 54], Giusti et al. have managed to put the linear covariant gauges on the lattice. This has allowed a numerical investigation of the transverse as well as of the longitudinal component of the gluon propagator. Following [54], let us introduce the transverse and longitudinal form factors $D_T(q)$ and $D_L(q)$ through

$$\left\langle A^a_\mu(q) A^b_\nu(-q) \right\rangle = \delta^{ab} \left(\frac{D_T(q)}{q^2} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{D_L(q)}{q^2} \frac{q_\mu q_\nu}{q^2} \right). \tag{5.1}$$

The results obtained in [52 – 54] show that both $D_T(q)$ and $D_L(q)$ are suppressed in the low momentum region, see for instance Fig.3 and Fig.4 of [54]. Our results are in qualitative agreement with the lattice data. Indeed, from the expression (4.19), we obtain

$$\begin{aligned} D_T(q) &= \frac{q^4}{q^4+m^2q^2+\gamma^4} . \\ D_L(q) &= \frac{\alpha q^2}{q^2+\alpha m^2} . \end{aligned} \tag{5.2}$$

exhibiting infrared suppression. Notice that, at least within the approximation considered in the present work, the suppression of the longitudinal form factor $D_L(q)$ in eq. (5.2) is a consequence of the dynamical gluon mass, due to the gluon condensate $\langle A^a_\mu A^a_\mu \rangle$, as already pointed out in [37]. Concerning now the ghost propagator and the Green function $\mathcal{G}_{tr}(k)$, to our knowledge, no results from lattice data are available so far.

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5.2 Comparison with the results obtained from the Schwinger-Dysons equations

The infrared behavior of the gluon and ghost propagator has been investigated within the Schwinger-Dyson framework in [55]. Here, a power-law *Ansatz* for the transverse and longitudinal form factors of the gluon propagator as well as for the ghost form factor $D_{gh}(q)$ has been employed, according to

$$\begin{aligned} D_T(q) &\approx \left(q^2\right)^{\sigma} . \\ D_L(q) &\approx \left(q^2\right)^{\rho} . \end{aligned} \tag{5.3}$$

and

$$\begin{aligned} \left\langle \overline{c}^a(q)c^b(-q)\right\rangle &= \delta^{ab}\frac{D_{gh}(q)}{q^2} . \\ D_{gh}(q) &\approx \left(q^2\right)^{\beta} . \end{aligned} \tag{5.4}$$

The results obtained for the infrared exponents (σ, ρ, β) turn out to be similar to those of the Landau gauge, namely[2]

$$\begin{aligned} \sigma &> 0 . \\ \rho &> 0 . \\ -\beta &= \frac{\sigma}{2} = \frac{\rho}{2} . \end{aligned} \tag{5.5}$$

indicating an infrared suppression of the transverse and longitudinal gluon form factors, and an infrared enhancement of the ghost propagator. Concerning the gluon propagator, these results are in qualitative agreement with our results as well as with the lattice data. However, concerning the ghost propagator, we have found that, instead of the ghost form factor, the quantity which is enhanced in the infrared is $\mathcal{G}_{tr}(k)$. For a better understanding of this point, it is worth reminding here that the result for the infrared exponents in eq. (5.5) has been obtained by using a bare-vertex truncation scheme [55]. This approximation has been proven successful in the Landau gauge [23 − 25, 27 − 29]. In particular, in the Landau gauge, no qualitative difference has been found if bare vertices are replaced by vertices dressed according to the Slanov-Taylor identities. This feature of the Landau gauge is believed to be deeply related to the nonrenormalization theorem of the ghost-antighost-gluon vertex, which holds to all orders of perturbation theory [56, 57]. Recently, the nonrenormalization theorem of the ghost-antighost-gluon vertex in the Landau gauge has been investigated through lattice simulations in [58], which have provided indications of its validity beyond perturbation theory. However, to our knowledge, no such a theorem is available in linear covariant gauges, for a nonvanishing value of the gauge parameter α. Furthermore, according to the authors [55], it is yet an open question whether the values of the infrared exponents in eq. (5.5) remain unchanged if bare vertices are replaced by dressed ones. Our results suggest that a different behavior might be expected when dressed vertices would be employed.

[2]The explicit values of these infrared exponents as well as their dependence from the gauge parameter can be found in [55].

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6. Conclusion

In this work we have attempted at analyzing the effects of the Gribov copies on the gluon propagator in linear covariant gauges. By considering small values of the gauge parameter α, a few properties of the Gribov copies have been established, allowing us to investigate the infrared behavior of the gluon two-point function.

As in the case of the Landau gauge, it turns out that the transverse component of the gluon propagator is suppressed in the infrared. Moreover, the longitudinal part is left unchanged, as shown in eq. (3.15). The infrared behavior of the gluon propagator has been investigated also in the presence of the gluon condensate $\langle A^a_\mu A^a_\mu \rangle$. In this case, the infrared suppression of the transverse component is enforced. Furthermore, its longitudinal component turns out to be suppressed as well, as expressed by eq. (4.19). These results are in qualitative agreement with those obtained from lattice simulations and from the analysis of the Schwinger-Dyson equations.

Concerning now the behavior of the ghost fields in linear covariant gauges, the output of our analysis is that, instead of the ghost propagator, the Green function which exhibits infrared enhancement is given by $\mathcal{G}_{tr}(k)$, as defined in eq. (1.5). It should be remarked that $\mathcal{G}_{tr}(k)$ does not coincide with the ghost propagator for a generic value of the gauge parameter α. However, $\mathcal{G}_{tr}(k)$ reduces to the ghost two-point function for vanishing α, so that our results allow to recover those of the Landau gauge in the limit $\alpha \to 0$.

Needless to say, many aspects of the covariant linear gauges remain still to be investigated. A partial list of them is:

As already pointed out, a suitable auxiliary functional corresponding to the linear covariant gauge fixing condition is not yet at our disposal. As in the case of the Landau gauge [5, 6, 31], this functional could be very helpful for a characterization of the properties of the Gribov copies not attainable by infinitesimal gauge transformations.

From the present analysis, it emerges that the Green function $\mathcal{G}_{tr}(k)$ has a special role, as it obeys the Gribov-Zwanziger horizon condition and reduces to the ghost two-point function in the Landau gauge. Although its dependence from the transverse component A^{aT}_μ of the gauge field suggests that it might have a deeper meaning, it would be worth to have a better understanding of $\mathcal{G}_{tr}(k)$.

It would be useful to have a consistent framework to compute quantum corrections to the gluon propagator and to $\mathcal{G}_{tr}(k)$. This would amount to construct a local renormalizable action in linear covariant gauges incorporating the effects of the Gribov copies, as done by Zwanziger in the Landau gauge [4, 7].

Finally, it would be interesting to have more data in the linear covariant gauges from lattice simulations on the gluon and ghost propagators as well as on the Green function $\mathcal{G}_{tr}(k)$.

Acknowledgments

The Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brazil), the Faperj, Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro, the SR2-UERJ and the Coordenação de Aperfeiç Nível Superior (CAPES) are gratefully acknowledged for financial support.

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